Packing number of a signed graph (G, σ)

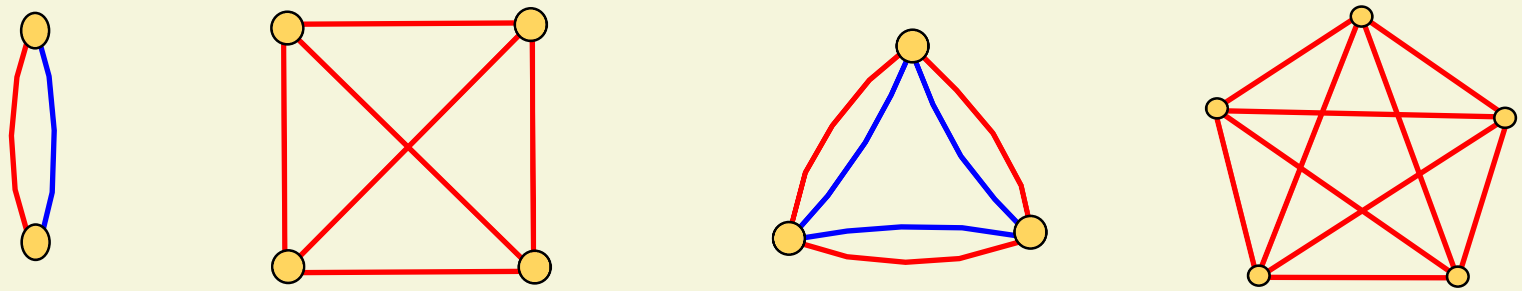
(signature)

Maximum number of switching equivalent signatures $\sigma_1, \sigma_2, \dots, \sigma_k$ no two of which assign negative sign to a same edge.

Notation: $P(G, \sigma)$

Note: $P(G, +) = \infty$

Examples



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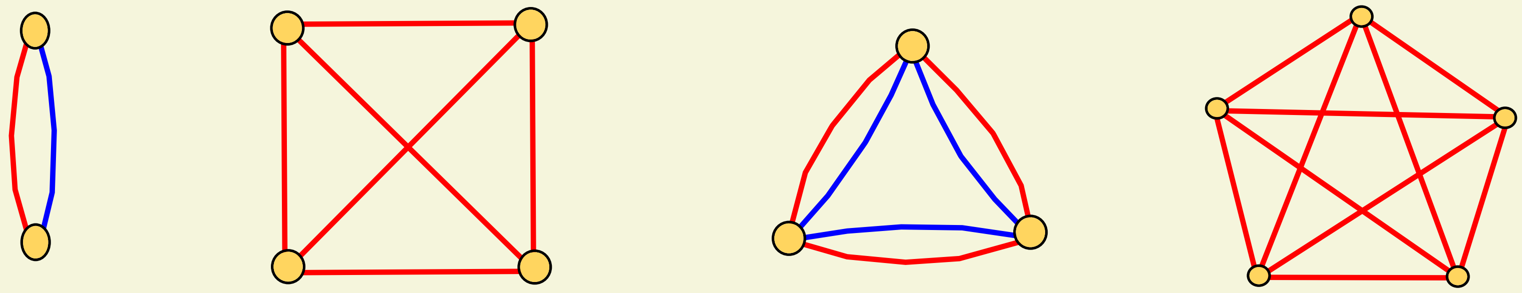


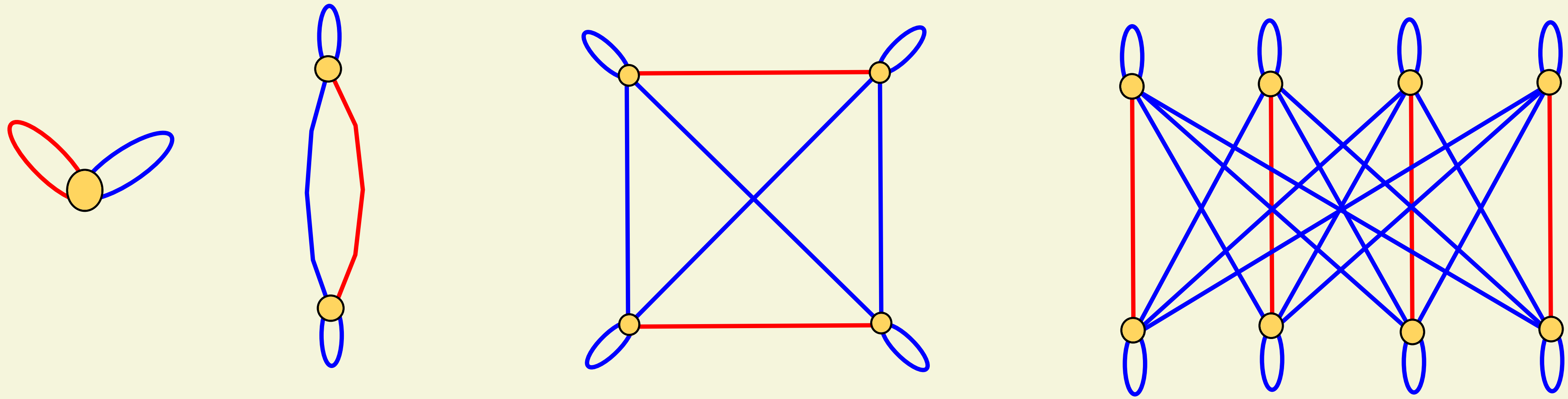
Lemma. $P(G, \sigma) \leq g_-(G, \sigma)$.

When equality holds we say (G, σ) packs.

Observation 1. $P(G, -)$ is always an odd number.

Observation 2. If G is bipartite, then $P(G, 6)$ is alway an even number.

Definition. $SPC^{\circ}(d)$ is obtained from $SPC(d)$ by adding a positive loop to each vertex.



Theorem. $P(G,\sigma) \geq d+1 \iff (G,\sigma) \to SPC^{\circ}(d)$

Proof of the theorem.

special case: If $\hat{G}$ is connected and has no positive odd closed walk, then $P(\hat{G})$ is the largest d such that

$$\hat{G} \to SPC(d).$$

Corollary. A graph G is 4-colorable if and only if $P(G,-) \geq 3$.

(equivalently $P(G,-) > 1$)

Corollary. A signed graph (G,σ) is $\{\pm 1, \pm 2\}$-colorable if and only if $P(G,-\sigma) \geq 2$.

Conjecture. Every signed planar graph with no positive odd walk packs.

negative cycle cover v.s. signature

A set of edges that covers every negative cycle.

Observation: Every equivalent signature is a negative cycle cover.

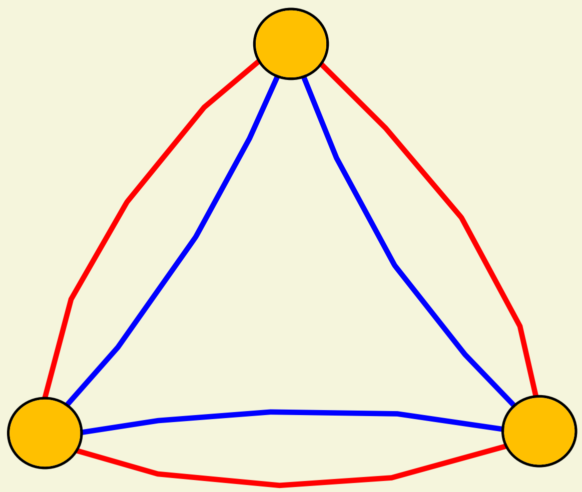
Theorem. Every minimal negative cycle cover is a

[Harry 1959] (minimal) signature.

Theorem. If (G,σ) has no K_3^2-minor, then the maximum number of edge-disjoint negative cycle covers is equal to the length of a shortest negative cycle.

[Gan, Jhonson]

K_3^2



Restating the theorem:

Every signed graph with no K_3^2-minor packs.

Theorem. Let $\mathcal{C}$ be a minor closed class of 4-colorable graphs. Let G be a bipartite graph in the class and let σ be any signature on G. Then $P(G, \sigma) \geq 4$.

Note 1. $P(G,-) \geq 3$ for every G in $\mathcal{C}$.

Note 2. $\mathcal{C}$ is included in the class of K_5-minor free graphs.

Note 3. The number of edges of each bipartite graph in $\mathcal{C}$ is at most $2n-4$. (average degree strictly less than 4)